cascade. However, if the excited nucleus rotates, any preferential location of "hot spots," say in the forward hemisphere, will be lost before decay. For a value of $J=10$, the time to rotate 180° is $\approx 3\times 10^{-20}$ sec and this is then the order of an upper limit for the fragmentation process.

We feel that the mechanism for fragmentation proposed by Wolfgang *et al.* is basically in agreement with the present observations. Fragmentation occurs during or shortly after the development of the nuclear cascade and before equipartition of energy is established. The fragments arise from regions of the nucleus which are highly disturbed ("hot spots"). The present experiment indicates that these excited regions are more concentrated in the forward hemisphere of the nucleus. The energies of most of the fragments are less than Coulombic, which may indicate large deviations from spherical shape at the moment of fragment formation.

We agree with Crespo *et al.*[4] that meson production, scattering, and reabsorption are not *necessary* for fragmentation and that Wolfgang *et al.*[1] may have overemphasized their role. Meson production, scattering and reabsorption in proton induced cascades will certainly increase the probability for the creation of highly disturbed regions and subsequent fragmentation. So also may bombardment with the correlated nucleons in an alpha particle as was observed by Crespo *et al.*

ACKNOWLEDGMENTS

The authors wish to thank Miss E. Norton for performing the chemical yield analyses and R. Withnell for target preparation. We are indebted to Dr. J. Alexander, Dr. G. Friedlander, Dr. J. R. Grover, Dr. S. Katcoff, Dr. M. L. Perlman, Dr. N. Sugarman, and Dr. A. Turkevich for many interesting discussions of our results and conclusions.

PHYSICAL REVIEW VOLUME 134, NUMBER 1B 13 APRIL 1964

Pseudoscalar Charge Density of Spin-$\frac{1}{2}$ Particles. I. Existence*

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(Received 4 November 1963)

When interactions are renormalizable and are invariant under time reversal but not invariant under space reflection, then under the requirement that the S matrix is free from divergences after renormalization it is shown that any spin-$\frac{1}{2}$ particle with nonvanishing mass should have pseudoscalar charge density in addition to the usual scalar charge density. Unrenormalizable interactions are also discussed as possible sources of the pseudoscalar charge density. Arguments are given for the observability of the pseudoscalar charge density.

1. INTRODUCTION AND DERIVATION OF THE PSEUDOSCALAR CHARGE DENSITY

THE purpose of this work is to study the electromagnetic properties spin-$\frac{1}{2}$ particles possess as a result of parity-nonconserving but time-reversal invariant interactions. The purpose of this section is to show that any charged spin-$\frac{1}{2}$ particle has pseudoscalar charge density in addition to usual scalar charge density as a result of the parity-nonconserving interactions. To show this, let us consider the Lagrangian density

$$L=L_1+L_2,\quad L_1=-:\bar{\psi}(x)\left[\gamma_\mu\frac{\partial}{\partial x_\mu}+m_0\right]\psi(x): \quad (1.1)$$

for a spin-$\frac{1}{2}$ field ψ in the Heisenberg representation, where the notation $:X:$ means to take the normal product of the operators included in X, ψ^* is the Hermitian conjugate of ψ, $\bar{\psi}=\psi^*\beta$, and γ_μ is a 4×4 Hermitian matrix and satisfies the commutation relation

$$\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}. \quad (1.2)$$

The interaction Lagrangian density L_2 is always assumed to be invariant under time reversal. To prove unambiguously that the pseudoscalar charge density exists, L_2 is assumed (for the moment) to include only renormalizable interactions.

The next step is to renormalize the wave function ψ for the free dressed particle with moment p interacting with its self-field as

$$\psi(p)=Z_2^{1/2}\psi_I(p), \quad (1.3)$$

where ψ_I represents the wave function in interaction representation and the c number Z_2 is a positive definite constant. Since the term $\bar{\psi}\gamma_\mu\gamma_5(\partial/\partial x_\mu)\psi$ is invariant under time reversal but the term $\bar{\psi}\gamma_5\psi$ is not, the former term as well as the self-energy term $\bar{\psi}\psi$ should be induced by the self-interaction of any spin-$\frac{1}{2}$ particle with finite mass, where $\gamma_5^2=1$. To renormalize as (1.3) shows, therefore, both the parity-nonconserving counter

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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term $\bar{\psi}\gamma_\mu\gamma_5(\partial/\partial x_\mu)\psi$ and the self-energy counter term $\bar{\psi}\psi$ should be added to the free part of the Lagrangian density L_1 and the same terms should be subtracted from the interaction Lagrangian density L_2.[1]

Then our starting Lagrangian density has the form

$$L=L_0+L',$$

$$L_0=-:\bar{\psi}(x)\left[\Gamma_\mu\frac{\partial}{\partial x_\mu}+m\right]\psi(x):, \qquad (1.4)$$

$$L'=L_2+\delta m:\bar{\psi}(x)\psi(x):+\frac{a}{(1-a^2)^{1/2}}:\bar{\psi}(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi(x):,$$

where a is a real constant by the requirement that L_2 is invariant under time reversal, and $a^2<1$ by the requirements that the free dressed particle has a nonvanishing mass and it cannot propagate in vacuum faster than the velocity of light. The definition of Γ_μ in L_0 is

$$\Gamma_\mu=\gamma_\mu(1+a\gamma_5)/(1-a^2)^{1/2}, \qquad (1.5)$$

which satisfies a commutation relation of the same form as Eq. (1.2) for γ_μ:

$$\{\Gamma_\mu,\Gamma_\nu\}=2\delta_{\mu\nu}. \qquad (1.6)$$

It should be noted that the definition of $\bar{\psi}$ in L_0 is still $\psi^*\beta$, not $\psi^*\beta(1+a\gamma_5)(1-a^2)^{-1/2}$, because the term $\gamma_\mu\gamma_5$ in Γ_μ comes from the self-interaction. When the definition $\bar{\psi}=\psi^*\beta(1+a\gamma_5)(1-a^2)^{-1/2}$ is incorrectly used instead, the Lagrangian density L_0 is not a Hermitian operator and it cannot be the free part of the Lagrangian density L.

From the Lagrangian density (1.4), the unrenormalized modified propagator of the spin-$\frac{1}{2}$ particle with nonvanishing mass is given [Eq. (15) of I] by

$$S_F'(p)=i\left\{-\delta m-\frac{a^2m}{1-a^2}+Z\int_0^\infty dx^2[(m-x)\sigma_1(x^2)+\rho_2(x^2)]\right\}S_F^2(p)+\left\{\frac{a}{1-a^2}+Z\int_0^\infty dx^2\rho_3(x^2)\right\}S_F(p)(\Gamma\cdot p)\gamma_5S_F(p)$$
$$+\left\{1-\frac{a^2}{1-a^2}-Z\int_0^\infty dx^2\sigma_1(x^2)\right\}S_F(p)-iZ\int_0^\infty dx^2\frac{(i\Gamma\cdot p-x)\sigma_1(x^2)+\rho_2(x^2)+i(\Gamma\cdot p)\gamma_5\rho_3(x^2)}{p^2+x^2-i\epsilon}, \qquad (1.7)$$

where $S_F(p)=i/(i\Gamma\cdot p+m)$, spectral functions σ_1, ρ_2, and ρ_3 do not include the $\delta(x^2-m^2)$ function, and Z is a real constant. The integrals $\int_0^\infty dx^2\sigma_1(x^2)$, $\int_0^\infty dx^2(1/x)\rho_2(x^2)$, and $\int_0^\infty dx^2\rho_3(x^2)$ diverge logarithmically in perturbation calculations. To determine the renormalization constants δm, a, and Z_2, we shall use the requirement (1.3) and the relation

$$S_F'(p)=Z_2S_{F,\mathrm{re}}'(p), \qquad (1.8)$$

where $S_{F,\mathrm{re}}'(p)$ is the renormalized modified propagator.

Then the renormalization constant Z_2 for the wave function is expressed as

$$Z_2^{-1}=C(1-a^2)/(1-3a^2), \qquad (1.9)$$

where $C\geqslant 1$. By the definition (1.3), Z_2 should have the meaning of a probability.[2] This means that

$$a^2\leqslant\tfrac{1}{3}. \qquad (1.10)$$

The renormalized modified propagator $S_{F,\mathrm{re}}'(p)$ can be expressed in terms of three spectral functions in the form

$$S_{F,\mathrm{re}}'(p)=-i\int_0^\infty dx^2\frac{(i\Gamma\cdot p-x)\rho_1(x^2)+\rho_2(x^2)+i(\Gamma\cdot p)\gamma_5\rho_3(x^2)}{p^2+x^2-i\epsilon}, \qquad (1.11)$$

where $\rho_1(x^2)\equiv\delta(x^2-m^2)+\sigma_1(x^2)$.

As was the case when the two counter terms (the self-energy and the parity-nonconserving counter terms) were introduced, the renormalized S matrix is a function of a, which is bounded by the condition (1.10). Thus the S matrix is free from divergences. In this paper it will be shown that a is related to the pseudoscalar charge density of any spin-$\frac{1}{2}$ particle with nonvanishing mass, and it may be an observable. When the parity-nonconserving counter term is not introduced, as will be shown in the next section, the S matrix is not free from divergences even after the renormalization. Thus, it is necessary to introduce the parity-nonconserving counter term.

The reader may doubt the consistency between the two statements: (1) When the two counter terms are introduced, the renormalized S matrix is a function of a and the matrix is divergence free. (2) When only the self-energy counter term is introduced, the S matrix is not free from divergences even after the

[1] K. Hiida, Phys. Rev. **132**, 1239 (1963). Hereafter this article will be cited as I.

[2] As was shown by Eq. (18) of I,

$$Z_2^{-1}=1+\int_0^\infty dx^2[\sigma_1-a\rho_3].$$

Using the inequality $\sigma_1-a\rho_3\geq 0$ leads to $1>Z_2\geq 0$.

renormalization. In the former case, the constant a is expressed as

$$a=\frac{1-\left\{1+4\left[Z_2\int_0^\infty dx^2\rho_3(x^2)\right]^2\right\}^{1/2}}{2\left[Z_2\int_0^\infty dx^2\rho_3(x^2)\right]}, \quad (1.12)$$

that is, the constant a is a function of $Z_2\int_0^\infty dx^2\rho_3(x^2)$. In perturbation calculations, Z_2^{-1} and the integral $\int_0^\infty dx^2\rho_3(x^2)$ diverge logarithmically. Thus, there is a possibility that $Z_2\int_0^\infty dx^2\rho_3(x^2)$ remains finite. In fact, the condition (1.10) requires

$$\left|Z_2\int_0^\infty dx^2\rho_3(x^2)\right|\leqslant\tfrac{1}{2}\sqrt{3}.$$

In the latter case, as will be shown in the next section, the renormalized S matrix is a function of the divergent integral $\int_0^\infty dx^2\rho_3(x^2)$, not of the convergent integral $Z_2\int_0^\infty dx^2\rho_3(x^2)$. Thus there is no inconsistency between the two statements.

Because of the gauge invariance of the theory, even if parity is not conserved, the differential operator $-\partial/\partial p_\mu$ is the operator to insert a photon vertex without momentum transfer into the part of a Feynman diagram that represents the propagation of any charged particle. Consequently, the Ward identity[3]

$$Z_1=Z_2$$

holds, where Z_1 is the renormalization constant for the photon vertex of any charged spin-$\frac{1}{2}$ particle. The renormalized photon vertex without momentum transfer on the mass shell for any charged spin-$\frac{1}{2}$ particle is given by

$$\bar\psi_I(p)S_F^{-1}(p)\left[-\frac{\partial}{\partial p_\mu}S_{F,\mathrm{re}}'(p)\right]S_F^{-1}(p)\psi_I(p)=\bar\psi_I(p)\Gamma_\mu\psi_I(p). \quad (1.13)$$

This equation means that the charge density of a free charged spin-$\frac{1}{2}$ particle is given by

$$\rho(x)=e:\psi_I^*(x)\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}\psi_I(x):, \quad (1.14)$$

which consists of the scalar part $(1-a^2)^{-1/2}$ and the pseudoscalar part $a\gamma_5(1-a^2)^{-1/2}$. We shall call the pseudoscalar part $e\psi^*a\gamma_5(1-a^2)^{-1/2}\psi$ the "pseudoscalar charge density."

It should be stressed that the pseudoscalar charge density is not arbitrarily introduced in this theory but is induced by parity-nonconserving interactions and, as will be discussed in Sec. 4, the constant a may be an observable when the charged particle is not free. The independent of a. From Eq. (A9) of I, the total charge of a charged spin-$\frac{1}{2}$ particle is given by

$$Q=\int d^3x\rho(x)=e\sum_\mu\int d^3k\{a_\mu^*(\mathbf{k})a_\mu(\mathbf{k})-b_\mu^*(\mathbf{k})b_\mu(\mathbf{k})\},$$

where $\sum_\mu$ denotes the summation over all possible spin states, and a_μ^* and b_μ^* are the particle and antiparticle creation operators, respectively.

At the end of Sec. 2, unrenormalizable interactions as the source of the pseudoscalar charge density are also discussed. The most general form of the photon vertex on the mass shell for any spin-$\frac{1}{2}$ particle with nonvanishing mass is given in Sec. 3. This form shows that a neutral spin-$\frac{1}{2}$ particle with nonvanishing mass also should have the pseudoscalar charge density. In order to help to understand our renormalization method and the origin of the pseudoscalar charge density, an example is also discussed in Sec. 3. Section 4 is devoted to a discussion of the observability of the pseudoscalar charge density.

2. ON THE UNIQUENESS OF THE EXISTENCE OF THE PSEUDOSCALAR CHARGE DENSITY

In Sec. 1 it was argued that when both the self-energy and the parity-nonconserving counter terms are introduced, the renormalized S matrix is free from divergences and, as the direct consequence of introducing the latter counter term, any charged spin-$\frac{1}{2}$ particle should have pseudoscalar charge density in addition to the usual scalar charge density. The purpose of this section is to show that when only the self-energy counter term is introduced, the S matrix is not free from divergences even after the renormalization.

To show this we shall start from the partially renormalized modified propagator

$$\hat S_{F,\mathrm{re}}'(p)=\int_0^\infty dx^2\rho_3(x^2)S_F(p)(\gamma\cdot p)\gamma_5S_F(p)-i\int_0^\infty dx^2\frac{(i\gamma\cdot p-x)\rho_1(x^2)+\rho_2(x^2)+i(\gamma\cdot p)\gamma_5\rho_3(x^2)}{p^2+x^2-i\epsilon}, \quad (2.1)$$

which is obtained from Eq. (1.7) by taking $a=0$, $Z=Z_2$, $\delta m=Z\int_2^\infty dx^2[(m-x)\sigma_1(x^2)+\rho_2(x^2)]$, and $Z_2^{-1}=\int_0^\infty dx^2\rho_1(x^2)$. When $a=0$, the expression (1.7) includes two constants, δm and Z, and three divergent integrals, $\int_0^\infty dx^2[(m-x)\sigma_1(x^2)+\rho_2(x^2)]$, $\int_0^\infty dx^2\rho_3(x^2)$, and $\int_0^\infty dx^2\sigma_1(x^2)$, which have coefficients with different transformation properties. Therefore, at least one divergent integral remains in the renormalized modified propagator $\hat S_{F,\mathrm{re}}'$ even after determining the magnitudes of the constants properly. We have so renormalized in Eq. (2.1) that, when parity is conserved, the present renormalization method coincides with Dyson's prescription.[4] When we want to renormalize such that

[3] J. C. Ward, Phys. Rev. **78**, 182 (1950).

[4] F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949).

the divergent term $\int_0^\infty dx^2\rho_3(x^2)$ disappears from Eq. (2.1), the renormalized S matrix is not free from the divergences which appear in the usual mass or charge renormalization.

If the divergent integral $\int_0^\infty dx^2\rho_3(x^2)$ in (2.1) is harmless for all renormalized S-matrix elements, we cannot reject the above renormalization prescription. We shall show such harmless examples. Differentiating $\hat{S}_{F,\mathrm{re}}'(p)$ with respect to p_μ, $p_\mu p_\nu$, etc., yields the exact matrix elements describing the interactions of a charged spin-$\frac{1}{2}$ particle with zero-momentum photons. The results are

$$\bar{\psi}_I(p)S_F^{-1}(p)\left[-\frac{\partial}{\partial p_\mu}\hat{S}_{F,\mathrm{re}}'(p)\right]S_F^{-1}(p)\psi_I(p)=\bar{\psi}_I(p)\gamma_\mu\psi_I(p),$$

$$\bar{\psi}_I(p)S_F^{-1}(p)\left[\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial p_\nu}\hat{S}_{F,\mathrm{re}}'(p)\right]S_F^{-1}(p)\psi_I(p)=\bar{\psi}_I(p)[\gamma_\mu S_F(p)\gamma_\nu+\gamma_\nu S_F(p)\gamma_\mu]\psi_I(p),\quad(2.2)$$

etc. In these examples, the divergent integral $\int_0^\infty dx^2\rho_3(x^2)$ appearing in Eq. (2.1) does not contribute to the results, and the pseudoscalar charge density derived in Sec. 1 also disappears. This is the direct consequence of the gauge invariance of the theory. Recently, Carhart[5] calculated the radiative correction to the photon-electron vertex in the lowest order of the weak-coupling constant, and showed that the divergent integral does not contribute to the photon-electron vertex in the lowest order. If all other S-matrix elements also are free from the divergent integral as shown in Eq. (2.2), our derivation of the pseudoscalar charge density is not unique. In the following, as an example, we shall show that the S-matrix element for a $\gamma-\pi^0$ process is not free from the divergent integral.

Because of our assumption of time-reversal invariance, the most general form of the renormalized meson-vertex operator $\Gamma_5(p_1,p_2)$ is

$$\Gamma_5(p_1,p_2)=\gamma_5f_1+\frac{(i\gamma\cdot p_1-i\gamma\cdot p_2)}{m}f_2+\frac{1}{2m}[(i\gamma\cdot p_1+m)\gamma_5+\gamma_5(i\gamma\cdot p_2+m)]f_3+\frac{1}{m^2}(i\gamma\cdot p_1+m)\gamma_5(i\gamma\cdot p_2+m)f_4,\quad(2.3)$$

where f_i ($i=1$, 2, 3, and 4) are functions of p_1^2, p_2^2, and $q^2=(p_1-p_2)^2$, and

$$f_1(p_1^2=-m^2,\ p_2^2=-m^2,\ q^2=0)=1.\quad(2.4)$$

When the parity-nonconserving counter term is introduced, γ matrices in Eq. (2.3) should be replaced by Γ's and all scalar functions f_i are divergence free. When it is not introduced, on the other hand, these functions may include divergences.

Now we shall calculate the S-matrix element for the $\gamma-\pi^0$ process $\gamma+x\to\pi^0+x$, where x is a charged spin-$\frac{1}{2}$ particle on the mass shell, the momentum of the photon is equal to zero, and the neutral pion is in a virtual state. The matrix element is given by

$$\bar{\psi}_I(p_1)S_F^{-1}(p_1)\left[-\left(\frac{\partial}{\partial p_{1\mu}}+\frac{\partial}{\partial p_{2\mu}}\right)\hat{S}_{F,\mathrm{re}}'(p_1)\Gamma_5(p_1,p_2)\hat{S}_{F,\mathrm{re}}(p_2)\right]S_F^{-1}(p_2)\psi_I(p_2)$$
$$=\bar{\psi}_I(p_1)\Bigg\{\left[f_1-2f_2\left(\int_0^\infty dx^2\rho_3(x^2)\right)-\tfrac{1}{4}(f_1+2f_3+4f_4)\left(\int_0^\infty dx^2\rho_3(x^2)\right)^2\right][\gamma_\mu S_F(p_1)\gamma_5+\gamma_5S_F(p_2)\gamma_\mu]$$
$$+2\left[f_1-f_2\left(\int_0^\infty dx^2\rho_3(x^2)\right)\right]\left(\int_0^\infty dx^2\rho_3(x^2)\right)[\gamma_\mu\gamma_5S_F(p_1)\gamma_5+\gamma_5S_F(p_2)\gamma_\mu\gamma_5]$$
$$+[f_1+f_3]\left(\int_0^\infty dx^2\rho_3(x^2)\right)^2[\gamma_\mu\gamma_5S_F(p_1)+S_F(p_2)\gamma_5\gamma_\mu]\Bigg\}\psi_I(p_2).\quad(2.5)$$

Since the right-hand side of Eq. (2.5) consists of three terms whose transformation properties differ from one another, the matrix element is divergence free when and only when each term is divergence free. Perturbation calculation shows that, for example, (f_1+f_3) is not identically zero. Thus it is necessary to introduce the parity-nonconserving counter term, and consequently the existence of the pseudoscalar charge density is proved under the requirement that the renormalized S matrix is free from divergences.

Equation (2.5) was obtained by introducing the self-energy counter term alone, and the scattering matrix diverges because it includes the divergent integral $\int_0^\infty dx^2\rho_3(x^2)$. On the other hand, when both the self-energy and the parity-nonconserving counter terms are introduced, the renormalized S matrix is free from the divergence.

[5] R. A. Carhart (unpublished).

In fact, the matrix element in Eq. (2.5) is also given by

$$\bar{\psi}_I(p_1)S_F^{-1}(p_1)\left[-\left(\frac{\partial}{\partial p_{1_\mu}}+\frac{\partial}{\partial p_{2_\mu}}\right)S_{F,\mathrm{re}}'(p_1)\Gamma_5(p_1,p_2)S_{F,\mathrm{re}}'(p_2)\right]S_F^{-1}(p_2)\psi_I(p_2)$$
$$=f_1(-m^2,\,-m^2,\,q^2)\bar{\psi}_I(p_1)[\Gamma_\mu S_F(p_1)\gamma_5+\gamma_5 S_F(p_2)\Gamma_\mu]\psi_I(p_2)\,, \quad (2.6)$$

which is free from the divergent integral $\int_0^\infty dx^2\rho_3(x^2)$, because the constant a in Γ_μ is a function of the finite integral $Z_2\int_0^\infty dx^2\rho_3(x^2)$, not of the divergent integral. Thus, it follows that the two statements mentioned in the previous section are not inconsistent.

So far we have considered only renormalizable interactions. However, weak interactions that are not invariant under space reflection are unrenormalizable. The interaction Lagrangian or Hamiltonian used customarily to describe weak decay phenomena is a phenomenological one; and when it is used to calculate radiative corrections, there is no consistent method of removing all divergences from observed quantities. However observed quantities should be finite and future developments should lead to a consistent theory in which all observed quantities are free from divergences. Since we are far from this goal at the present moment, we shall introduce a phenomenological cutoff momentum Λ to get finite results. When the cutoff momentum is introduced, all quantities appearing in our theory are finite but renormalization is still necessary whenever we start from a bare-particle state. Even in this case it is reasonable to require again the renormalization condition (1.3). When this condition is required, the existence of the pseudoscalar charge density is evident. People believe that the $K_1^0-K_2^0$ mass difference could be explained by weak unrenormalizable interactions between K_1^0 and K_2^0 and their self-fields. Likewise it is reasonable to believe that weak unrenormalizable interactions are once of the sources of the pseudoscalar charge density.

3. PHOTON VERTEX ON THE MASS SHELL

We want to get the most general form for the photon vertex of both neutral and charged spin-$\frac{1}{2}$ particles with finite mass on their mass shells. Under the requirements of covariance and time-reversal invariance the photon vertex is expressed as

$$\bar{\psi}(p_1)j_\mu(p_1,p_2)\psi(p_2)$$
$$=ie\bar{\psi}_I(p_1)\left\{\Gamma_\mu f_1(q^2)+\frac{i}{2m}p_\mu f_2(q^2)\right.$$
$$\left.+\Gamma_\mu\gamma_5 f_3(q^2)+\frac{2i}{m}q_\mu\gamma_5 f_4(q^2)\right\}\psi_I(p_2)\,, \quad (3.1)$$

where $p=(p_1+p_2)$, $q=(p_1-p_2)$, and the four form factors f_i are all real when q is a space-like vector. The identity

$$i\bar{\psi}_I(p_1)p_\mu\psi_I(p_2)=\bar{\psi}_I(p_1)[\sigma_{\mu\nu}q_\nu-2m\Gamma_\mu]\psi_I(p_2)\,,$$

where

$$\sigma_{\mu\nu}=\tfrac{1}{2}i[\Gamma_\mu\Gamma_\nu-\Gamma_\nu\Gamma_\mu]=\tfrac{1}{2}i[\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu]\,,$$

can be used to re-express the first two terms in (3.1) as

$$ie\bar{\psi}_I(p_1)\left\{\Gamma_\mu F_1(q^2)+\frac{1}{2m}\sigma_{\mu\nu}q_\nu F_2(q^2)\right\}\psi_I(p_2)\,, \quad (3.2)$$

where $F_1=f_1-f_2$ and $F_2=f_2$. The expression (3.2) is gauge invariant. The renormalization condition, Eq. (1.13), for the photon vertex leads to the conditions

$$F_1(0)=1 \text{ for charged particles}, \quad (3.3)$$
$$=0 \text{ for neutral particles},$$

and

$$f_3(q^2)=(q^2/m^2)F_3(q^2)\,,$$

where $|F_3(0)|<\infty$. The Dirac equation for $\bar{\psi}_I(p_1)$ and $\psi_I(p_2)$ leads to

$$i\bar{\psi}_I(p_1)q_\mu\gamma_5\psi_I(p_2)=\frac{1}{2m}\psi_I(p_1)(\Gamma\cdot q)q_\mu\gamma_5\psi_I(p_2)\,.$$

Therefore the last two terms in (3.1) are re-expressed as

$$\frac{ie}{m^2}\bar{\psi}_I(p_1)\{q^2\Gamma_\mu F_3(q^2)+(\Gamma\cdot q)q_\mu f_4(q^2)\}\gamma_5\psi_I(p_2)\,,$$

which is gauge invariant only when $f_4(q^2)=-F_3(q^2)$. This brings us to the gauge invariant and renormalized photon vertex

$$\bar{\psi}(p_1)j_\mu(p_1,p_2)\psi(p_2)$$
$$=ie\bar{\psi}_I(p_1)\left\{\Gamma_\mu F_1(q^2)+\frac{1}{2m}\sigma_{\mu\nu}q_\nu F_2(q^2)\right.$$
$$\left.+\frac{1}{m^2}[q^2\Gamma_\mu-(\Gamma\cdot q)q_\mu]\gamma_5 F_3(q^2)\right\}\psi_I(p_2)\,. \quad (3.4)$$

A very similar expression for the photon vertex was obtained by Zel'dovich and Perelomov.[6] The difference between their expression and our expression (3.4) is that the later satisfies Eq. (1.13) but their expression

[6] Ya. B. Zel'dovich and A. M. Perelomov, Zh. Eksperim. i Teor. Fiz. **39**, 115 (1960) [English transl.: Soviet Phys.—JETP **12**, 777 (1961)].

satisfies Eq. (2.2) when the transferred momentum q is equal to zero. When renormalizable interactions are considered, according to their definition of the photon vertex, the S matrix is not free from divergences. The first term in Eq. (3.4) consists of the vector current $\bar{\psi}\gamma_\mu(1-a^2)^{-1/2}F_1\psi$ and the induced pseudovector current $\bar{\psi}a\gamma_\mu\gamma_5(1-a^2)^{-1/2}F_1\psi$, which are related to the scalar charge density and the induced pseudoscalar charge density, respectively. The second and the third terms in Eq. (3.4) are related to the anomalous magnetic moment and the anapole moment.[7] It is evident from Eq. (3.4) that neutral spin-$\frac{1}{2}$ particles also have the pseudoscalar charge density.

The parity-nonconserving counter term includes the operator $(\partial/\partial x_\mu)$. For charged spin-$\frac{1}{2}$ particles, therefore, the parity-nonconserving counter term should be introduced in a gauge-invariant way. As an example, consider the system which involves the electron field ψ, the photon field A_μ, the electron-neutrino field ψ_ν, and a charged vector-meson field ϕ_μ interacting with ψ and ψ_ν. The gauge-invariant Lagrangian density of the system is given by

$$L=-:\bar{\psi}\left[\Gamma_\mu\left(\frac{\partial}{\partial x_\mu}-ie_0A_\mu\right)+m\right]\psi:-\tfrac{1}{2}:\left(\frac{\partial A_\nu}{\partial x_\mu}\right)\left(\frac{\partial A_\nu}{\partial x_\mu}\right):-:\bar{\psi}_\nu\gamma_\mu\frac{(1+\gamma_5)}{2}\frac{\partial}{\partial x_\mu}\psi_\nu:$$

$$-\left[\tfrac{1}{2}:G_{\mu\nu}{}^*G_{\mu\nu}:+M^2:\phi_\mu{}^*\phi_\mu:+ie_0\mu:F_{\mu\nu}\phi_\mu{}^*\phi_\nu:\right]+ig\left[\bar{\psi}\gamma_\mu\phi_\mu\frac{(1+\gamma_5)}{2}\psi_\nu+\bar{\psi}_\nu\gamma_\mu\phi_\mu{}^*\frac{(1+\gamma_5)}{2}\psi\right]$$

$$+\delta m:\bar{\psi}\psi:+a(1-a)^{-1/2}:\bar{\psi}\gamma_\mu\gamma_5\left(\frac{\partial}{\partial x_\mu}-ie_0A_\mu\right)\psi:+\delta M^2:\phi_\mu{}^*\phi_\mu:, \tag{3.5}$$

where

$$\Gamma_\mu=\gamma_\mu(1+a\gamma_5)(1-a^2)^{-1/2},\quad F_{\mu\nu}=\left(\frac{\partial A_\nu}{\partial x_\mu}-\frac{\partial A_\mu}{\partial x_\nu}\right),\quad G_{\mu\nu}=\left(\frac{\partial}{\partial x_\mu}-ie_0A_\mu\right)\phi_\nu-\left(\frac{\partial}{\partial x_\nu}-ie_0A_\nu\right)\phi_\mu.$$

μ is the anomalous magnetic moment of the vector meson in units of the vector-meson magneton, e_0 is the bare coupling constant for photon interactions, and m and M are the masses of the electron and the vector meson, respectively.

We shall explain our renormalization method for the photon vertex of the electron. For convenience sake, the diagrams representing the photon vertex of the electron are divided into two classes: (1) diagrams in the order $e_0{}^{2n}g^0$ for the radiative corrections to the vertex part and the external electron lines, and (2) diagrams in the order $e_0{}^{2n}g^{2(m+1)}$ and $e_0{}^{2n}g^{2m}a^{(l+1)}$ for the radiative corrections, where l, m, and n are positive integers including zero. As we know, the contribution from all diagrams that belong to the former class satisfies the renormalization condition (1.13). The constant a is already determined by Eq. (1.12) in terms of spectral functions appearing in the renormalized modified propagator $S_{F,\mathrm{re}}'(p)$ of the electron. From this value of a, it follows that the contribution from all diagrams that belong to the latter class is equal to zero when the momentum transfer q is equal to zero. This is the direct consequence of the gauge invariance of our Lagrangian (3.5).

Since the coupling constants e^2 and g^2 are very small, we shall consider only the order eg^2 for the diagrams belonging to the latter class. Then the possible diagrams are those shown in Fig. 1. We want to calculate the contribution from these diagrams to the parity-nonconserving part of the first term, and the second and the third terms in Eq. (3.4). The diagrams (c), (d), (e), and (f) contribute only to the wave-function renormalization constant of the

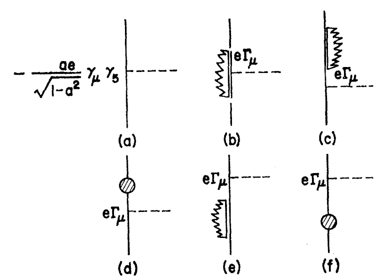

FIG. 1. Diagrams representing the photon vertex of the electron in the order eg^2 or ea. The full, dashed, wavy, and heavy lines denote the electron, the photon, the neutrino, and the vector meson, respectively. The circle on the electron line denotes the self-energy and the parity-nonconserving counter terms $[-i\delta m+a(1-a^2)^{-1/2}\gamma\cdot p_i\gamma_5]$, where p_1 and p_2 denote the momenta of the final and initial external electrons, respectively. The operators at each photon vertex are written explicitly in the diagrams.

[7] Ya. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. **33**, 1531 (1957) [English transl.: Soviet Phys.—JETP **6**, 1184 (1958)].

electron. From the diagrams (a) and (b), one gets

$$M_\mu=-\frac{ig^2}{2(2\pi)^4}\int d^4k\{\delta_{\alpha\beta}(p_1+p_2-2k)_\mu-(p_1-k)_\beta\delta_{\alpha\mu}-(p_2-k)_\alpha\delta_{\beta\mu}+(1+\mu)[q_\beta\delta_{\alpha\mu}-q_\alpha\delta_{\beta\mu}]\}$$
$$\times\bar\psi(p_1)\left[\gamma_\beta+\frac{1}{M^2}(\gamma\cdot p_1-\gamma\cdot k)(p_1-k)_\beta\right](\gamma\cdot k)\left[\gamma_\alpha+\frac{1}{M^2}(\gamma\cdot p_2-\gamma\cdot k)(p_2-k)_\alpha\right](1+\gamma_5)\psi(p_2)$$
$$\times[(k^2-2p_1k+M^2-m^2-i\epsilon)(k^2-i\epsilon)(k^2-2p_2k+M^2-m^2-i\epsilon)]^{-1}-a\bar\psi(p_1)\gamma_\mu\gamma_5\psi(p_2),\quad(3.6)$$

where the a^2 term is neglected. Since the k integration diverges quadratically, the cutoff factor of the type $\lambda^2/(p^2+\lambda^2-i\epsilon)$ is introduced for each vector-meson propagator. Retaining only the most divergent parts, one obtains

$$M_\mu'\approx\frac{mg^2}{(4\pi)^2M^2}\left\{-\frac{1}{2}\int_0^{\lambda^2}dL_1dL_2\int_0^1\frac{x(1-x)dx}{[L_1x+L_2(1-x)+M^2+x(1-x)q^2-i\epsilon]^2}\right.$$
$$\left.+\mu\int_0^{\lambda^2}dL\int_0^1\frac{y(1-y)dxdy}{[L(1-y)+M^2y-m^2y(1-y)+x(1-x)y^2q^2-i\epsilon]}\right\}\bar\psi(p_1)\sigma_{\mu\nu}q_\nu\psi(p_2)$$
$$+\frac{g^2}{2(2\pi)^2M^2}\int_0^{\lambda^2}dL_1dL_2\int_0^1\frac{x(1-x)dx}{[L_1x+L_2(1-x)+M^2+x(1-x)q^2-i\epsilon]}\bar\psi(p_1)\left\{3\gamma_\mu-\frac{\mu}{2M^2}[q^2\gamma_\mu-2\mathrm{Im}q_\mu]\right\}\gamma_5\psi(p_2)$$
$$+\frac{g^2}{2(4\pi)^2M^2}\int_0^{\lambda^2}dL\int_0^1\frac{xdx}{[Lx+M^2+x(1-x)q^2-i\epsilon]}\bar\psi(p_1)\{-2x(1-x)q^2\gamma_\mu+\mathrm{Im}[3-(1-2x)^2]q_\mu\}\gamma_5\psi(p_2)$$
$$-\frac{g^2}{(4\pi)^2M^2}\int_0^{\lambda^2}dL\int_0^1\frac{y(1-y)dxdy}{[L(1-y)+M^2y-m^2y(1-y)+x(1-x)y^2q^2-i\epsilon]}$$
$$\times\mathrm{Im}q_\mu\bar\psi(p_1)\gamma_5\psi(p_2)-a\bar\psi(p\cdot)\gamma_\mu\gamma_5\psi(p_2).\quad(3.7)$$

By comparing the second term in Eq. (3.4) with the first term in Eq. (3.7) and passing to the limit $\lambda^2\to\infty$, one finds

$$F_2^{(w)}(q^2)\approx-\frac{g^2}{(4\pi)^2}\frac{m^2}{M^2}(1-\mu)\int_{4M^2}^{\Lambda^2}dx^2\frac{1}{q^2+x^2-i\epsilon}\frac{(x^2-4M^2)^{1/2}}{x},$$

where the superscript (w) means that this expression is obtained by taking account only of weak interaction. Since this expression diverges, again the cutoff momentum Λ was introduced. We get

$$F_2^{(w)}(0)\approx-\frac{g^2}{(4\pi)^2}\frac{m^2}{M^2}(1-\mu)\ln\frac{\Lambda^2}{M^2}.\quad(3.8)$$

On the other hand, to the order $\alpha\equiv e^2/4\pi$ we get

$$F_2^{(\mathrm{e.m.})}(q^2)=\frac{\alpha}{\pi}\int_{4m^2}^{\Lambda^2}dx^2\frac{1}{q^2+x^2-i\epsilon}\frac{m^2}{x(x^2-4m^2)^{1/2}},$$

which gives

$$F_2^{(\mathrm{e.m.})}(0)\approx\frac{\alpha}{2\pi}-\frac{\alpha}{\pi}\frac{m^2}{\Lambda^2},\quad(3.9)$$

$$\langle r^2\rangle_2^{1/2}=\frac{1}{m},\quad(3.10)$$

where the superscript (e.m.) means that these expressions are obtained by electromagnetic interaction to the order α, and $\langle r^2\rangle_2^{1/2}$ denotes the root-mean-square radius of the anomalous magnetic moment of the electron, which is just equal to the Compton wavelength of the electron to the order α. Using the value $g^2/M^2\approx(2\sqrt{2}/M_N^2)\times10^{-5}$, where M_N is the nucleon mass, and assuming $\mu\approx1$, one finds that the values of $F_2(0)^{(w)}$ given by Eq. (3.8) and of the second term in Eq. (3.9) become comparable at $1/\Lambda\approx10^{-16}$ cm.

From the remaining terms in Eq. (3.7), one finds in the limit $\lambda^2\to\infty$ that

$$M_\mu'-\text{moment term}$$
$$\approx\left[\frac{1}{2}\int_0^{\Lambda^2}dx^2X(x^2)-a\right]\bar\psi(p_1)\gamma_\mu\gamma_5\psi(p_2)$$
$$-\frac{1}{6}\left[\frac{\mu^2}{2M^2}\int_0^{\Lambda^2}dx^2\frac{x^2X(x^2)}{q^2+x^2-i\epsilon}+5\int_0^{\Lambda^2}dx^2\frac{X(x)}{q^2+x^2-i\epsilon}\right]$$
$$\times\bar\psi(p_1)[q^2\gamma_\mu-\gamma\cdot qq_\mu]\gamma_5\psi(p_2),\quad(3.11)$$

where

$$X(x^2)=\theta(x^2-4M^2)\frac{g^2}{2(4\pi)^2M^2}\frac{(x^2-4M^2)^{1/2}}{x}.$$

The cutoff momentum Λ has again been introduced in expression (3.11). When the renormalization condition (1.13) is applied to the first term in Eq. (3.11), the constant a is found to be

$$a\approx\frac{1}{2}\int_0^{\Lambda^2}dx^2X(x^2)\approx\frac{g^2}{(8\pi)^2}\frac{\Lambda^2}{M^2}. \tag{3.12}$$

As was shown by Eq. (55) of I, this value of a coincides with that given by Eq. (1.12). From Eqs. (3.4) and (3.11), $F_3(0)$ is given by

$$F_3(0)\approx-\frac{g^2}{6(8\pi)^2}\mu\frac{m^2}{M^2}\frac{\Lambda^2}{M^2}-\frac{5}{3}\frac{g^2}{(8\pi)^2}\frac{m^2}{M^2}\ln\frac{\Lambda^2}{M^2}. \tag{3.13}$$

All our expressions for the electron also hold for the muon if m is the muon mass.

4. OBSERVABILITY OF THE PSEUDOSCALAR CHARGE DENSITY

We know the theorem[8]: Since two gamma matrices γ_μ and Γ_μ satisfy the same commutation relation, as was shown by Eqs. (1.2) and (1.6), there exists a nonsingular matrix S such that

$$\Gamma_\mu=S\gamma_\mu S^{-1} \tag{4.1}$$

and S is unique except for an arbitrary multiplicative factor. In our case S and its inverse are given by

$$\begin{aligned}S&=\left\{\frac{(1-a^2)^{1/2}}{2[1+(1-a^2)^{1/2}]}\right\}^{1/2}\left\{1+\frac{(1-a\gamma_5)}{(1-a^2)^{1/2}}\right\},\\ S^{-1}&=\left\{\frac{(1-a^2)^{1/2}}{2[1+(1-a^2)^{1/2}]}\right\}^{1/2}\left\{1+\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}\right\}.\end{aligned} \tag{4.2}$$

On the basis of the Lagrangian density

$$L_0=-:\bar{\psi}'(x)\left[\gamma_\mu\frac{\partial}{\partial x_\mu}+m\right]\psi'(x):, \tag{4.3}$$

it follows that the transformation $\psi=S\psi'$ leads to

$$L_0=-:\bar{\psi}(x)\left[\Gamma_\mu\frac{\partial}{\partial x_\mu}+m\right]\psi(x):, \tag{4.4}$$

where $\bar{\psi}=\psi^*\beta$. The reader may raise the following objections. Since the Lagrangian density (4.3) clearly conserves parity, so does the Lagrangian density (4.4). Therefore, referring to the term $[-a(1-a^2)^{-1/2}\bar{\psi}\gamma_\mu\gamma_5\times(\partial/\partial x_\mu)\psi]$ in the expression (4.4) as a parity-nonconserving term is misleading. Further demanding $a^2\leqslant\frac{1}{3}$ is incorrect because the constant a in the expression (4.2) can have any value between 1 and -1.

It is well known that two Lagrangian densities (4.3) and (4.4) with different representations of the Dirac gamma matrix describe the same free particle with spin $\frac{1}{2}$. As far as only the free spin-$\frac{1}{2}$ particle is concerned, the objection is valid. However, it was shown that the term $[-a(1-a^2)^{-1/2}\bar{\psi}\gamma_\mu\gamma_5(\partial/\partial x_\mu)\psi]$ is induced by parity-nonconserving self-interaction. Consequently, any charged spin-$\frac{1}{2}$ particle should have an induced pseudoscalar charge density and $a^2\leqslant\frac{1}{3}$ is obtained under the requirement that the wave-function renormalization constant Z_2 should have the meaning of a probability.[2] That is, these results were obtained by considering the interaction of the spin-$\frac{1}{2}$ particle with other fields; and these cannot be obtained by considering only the free particle without interaction— as can be seen from the Lagrangian densities (4.3) and (4.4). We may argue that the constant a should be an observable because it is finite and is uniquely determined by Eq. (1.12). One must be careful about the nonunitary nature of the transformation matrix S, i.e., of the fact that

$$S^*S=(1-a\gamma_5)/(1-a^2)^{1/2}\neq1 \quad\text{for}\quad a\neq0. \tag{4.5}$$

Because of this nonunitary nature of S, the constant a may be an observable when the particle interacts with other fields, even though it has no physical meaning when the particle is free.

In the next paper entitled "Pseudoscalar Charge Density of Spin-½ Particles. II. Observability," we shall estimate the influence of the pseudoscalar charge density on the spin orientation of charged spin-$\frac{1}{2}$ particles in electric and magnetic fields. For example, when an electron moves in a longitudinal electric field (electric field $\mathbf{E}$ is parallel to the momentum of the electron $\mathbf{p}$), its spin rotates about the axis $[\mathbf{p}\times\mathbf{M}]$ by the interaction of the pseudoscalar charge density with the longitudinal electric field, where $\mathbf{M}$ is the spin vector of the electron. Thus, it is clearly shown that the pseudoscalar charge density is an observable.

ACKNOWLEDGMENTS

The author would like to thank Dr. M. Hamermesh and Dr. M. Peshkin for their warm hospitality at the Argonne National Laboratory and Dr. F. E. Throw for careful reading of the manuscript.

[8] See, for example, J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), p. 425.